Mitigation of Correlated Non-Linearities in Digital Phased Arrays Using Channel-Dependent Phase Shifts

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***Abstract* — In an active phased array, each Transmitter/Receiver Module (TRM) performs a set of approximately linear functions (e.g., amplification, mixing, etc.) with the resulting signals later combined via beamforming techniques. Since these nearly-linear functions are performed prior to beamforming, it is theoretically possible to improve upon the dynamic range (DR) of each TRM through post-module array integration gain. It has been demonstrated [1], however, that DR enhancement may be limited by correlated nonlinear distortion (i.e., correlated from module to module).**

A general technique that ensures nonlinearities do not add constructively from module to module has been proposed recently [2], and verified experimentally for a special case [1]. Another special case of the general technique is described analytically in [3], but with no experimental verification. In this paper, we correct a flaw in the analysis presented in [3], and extend that analysis. Measurements on a thirteen channel digital phased array demonstrate that introducing random phase shifts into an array can substantially mitigate nonlinear distortion, thus improving DR over the array.

I. INTRODUCTION

Phased arrays are used in a wide variety of applications ranging from radar to sonar and wireless communications. Such arrays consist of many sensors (e.g., antenna elements or subarrays). To extract useful information from such arrays, the signal received by each sensor is typically amplified, filtered, demodulated and then combined with other sensors. The amplification, filtering, and demodulation functions are performed by a device called a receiver. Active arrays contain many such receivers, one behind each sensor. The subsequent combining of sensor data is performed by a beamformer.

Ideally, the various transmitter and receiver chains are comprised of linear functional components. However, the devices used to implement these functions are often only approximately linear. As a result, each module's output can contain undesirable distortion due to device nonlinearities.

After processing within each module, output signals (including distortion and noise) are combined via beamforming. This beamforming is typically designed to impart "array integration gain" onto signals of interest i.e., to increase the strength of the signal relative to background noise. Unfortunately, it has been demonstrated that device nonlinearities are often correlated from module to module and thus may also be subject to array integration gain [1]. In fact, even when the nonlinear distortion is buried below the noise floor at the output of each module, array integration gain can amplify the distortion, causing it to exceed the noise at the output of the beamformer, thus limiting DR.

In earlier papers [1]-[2], we described a technique to improve DR by forcing nonlinearities to decorrelate from module to module, and therefore to add incoherently in the beamformer. We presented a specific demonstration using offset LOs to decorrelate nonlinearities in each module. In this paper, we describe a thirteen-channel digital phased array receiver array used to measure correlation between modules, and used to validate the specific case of phase shifts within the general methodology for reducing nonlinear distortion.

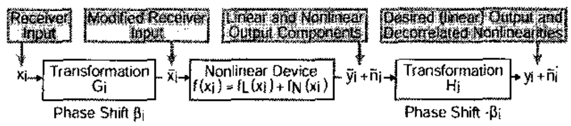

Fig. 1. General approach to mitigating nonlinearities in digital arrays using channel dependent transformations.

II. REVIEW OF GENERAL METHODOLOGY

For a phased array with N receivers, each receiver produces distortion that may be partially correlated and coherent across the array. To decorrelate the nonlinear distortion in a phased array, we perform two transformations on each receiver: one at the input and one at the signal output, but prior to beamforming. These transformations are unique to each receiver, and are chosen such that nonlinear distortion products created during analog processing are uncorrelated.

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As seen in Fig. 1, the first transformation modifies the input signal, x_i, to produce $\tilde{x}_i$. This modified input signal is then processed by nonlinear device(s), producing (modified) linear and nonlinear outputs. The second transformation is used to restore the linear output term y_i. The transformations G_i and H_i are chosen to restore the linear output without restoring the nonlinear output.

The key to this transformation process is that G_i and H_i are varied from receiver to receiver. This ensures that the nonlinear distortion at the output, $\tilde{n}_i'$, varies from receiver to receiver so the nonlinear elements will add incoherently in the beamformer. This point marks a drastic shift from traditional array receiver design techniques, which attempt to make receiver paths as identical as possible.

We used our experimental test bed to consider two forms of nonlinear distortion for potential mitigation: harmonic spurs and *MxN* mixer spurs. This use of phase shifts to mitigate nonlinear distortion is presented in [3], but with significant shortcomings that include errors in analysis. The analysis here is more thorough, and is verified experimentally.

III. PHASE SHIFTS AS TRANSFORMATION

We will show that phase shifts can be used as the input transformation to mitigate two types of nonlinearity: harmonic distortion and mixer *MxN* products. The analysis for harmonics is presented below. It yields the unfortunate result that fixed phase shifts cannot be used to mitigate third order intermodulation distortion. The analysis of mixer *MxN* products is beyond the scope of this paper. Experimental verification for both claims is presented later.

A. Phase Shifts and Harmonic Distortion

The mathematical notation in [3] is convenient for investigating the effects of a phase shift at harmonic frequencies, and we adopt it here. This will also make it easier to compare our results. We write the input to the r[th] receiver at frequency ω_0 in the form

$$v_r(t)=\left(\frac{1}{2}\right)\left[\boldsymbol{u}(t)e^{j(\omega_0 t+\theta_r+\beta_r)}+\boldsymbol{u}^*(t)e^{-j(\omega_0 t+\theta_r+\beta_r)}\right]$$

where $\boldsymbol{u}(t)=I(t)+jQ(t)$ is the baseband representation of the received signal, θ_r is a combination of the phase shifts on the signal due to both the beam direction and systematic phase shifts in the channel, and β_r is an additional phase shift applied to the r[th] receiver chain for the purpose of non-linearity mitigation. Note in the above equations and in what follows that boldfaced variables are complex. We can represent the output of the nonlinearity in the receiver by a power series as

$$\tilde{y}_r=f(v_r(t))=\sum_{k=0}^{\infty}b_k(v_r(t))^k$$

where the b_k are real constants. In general, the b_k will vary slightly from channel to channel; however, this variation can be ignored here with no loss of generality.

We now expand this power series using the binomial theorem, and rearrange terms to get

$$\tilde{y}_r(t)=2\sum_{m=1}^{\infty}\left[(\boldsymbol{u}(t))^m e^{jm(\omega_0 t+\theta_r+\beta_r)}+(\boldsymbol{u}^*(t))^m e^{-jm(\omega_0 t+\theta_r+\beta_r)}\right]$$
$$\times\sum_{i=0}^{\infty}b_{m+2i}\left(\frac{1}{2}\right)^{m+2i}\binom{m+2i}{i}|\boldsymbol{u}(t)|^{2i}+\sum_{i=0}^{\infty}b_{2i}\left(\frac{1}{2}\right)^{2i}\binom{2i}{i}|\boldsymbol{u}(t)|^{2i}$$

This expression is interpreted in the following way. The m index applies to the m^{th} harmonic, and we see that an applied phase shift of β_r at the input becomes a phase shift of $m\beta_r$ at the output. The m[th] harmonic is produced by the m[th] and higher order terms in the power series relating the output voltage to the input voltage. The inner sum captures these contributions, and for $m=1$, it represents intermodulation distortion caused by odd-order terms in the power series. The last term is the DC contribution from the even power terms in the power series.

To apply the inverse transformation, which is a phase shift of $-\beta_r$, to $y_r(t)$, we multiply the terms containing $\boldsymbol{u}(t)$ by $e^{-j\beta_r}$, and the terms containing $\boldsymbol{u}^*(t)$ by $e^{+j\beta_r}$, yielding:

$$y_r(t)=2\sum_{m=1}^{\infty}\left[(\boldsymbol{u}(t))^m e^{j[m(\omega_0 t+\theta_r)+(m-1)\beta_r]}+(\boldsymbol{u}^*(t))^m e^{-j[m(\omega_0 t+\theta_r)+(m-1)\beta_r]}\right]$$
$$\times\sum_{i=0}^{\infty}b_{m+2i}\left(\frac{1}{2}\right)^{m+2i}\binom{m+2i}{i}|\boldsymbol{u}(t)|^{2i}+\sum_{i=0}^{\infty}b_{2i}\left(\frac{1}{2}\right)^{2i}\binom{2i}{i}|\boldsymbol{u}(t)|^{2i}$$

The phase shift of β_r is removed from the fundamental signal ($m=1$).[1] This term includes in-band third (and higher) intermodulation distortion. However, a phase shift of $(m-1)\beta_r$ remains for the harmonics of the signal. Since each β_r is unique, the harmonic output of each receiver is uniquely phase shifted, enabling mitigation by phase shift.

B. Mitigation via Random Phase Shifts

Ideally, phase shifts would be chosen in a deterministic fashion to optimally mitigate addition of nonlinear distortion products. However, this may not be tractable for the general case of distortion created by a signal that could be received from all possible directions. We therefore consider the simpler case of random phase shifts applied to the signal in all channels.

By design, a random phase shift of β_i is added to the i^{th} channel. We assume β_i is a uniform random variable on $[0,2\pi]$. For either the case of an *MxN* mixer product or an m^{th} harmonic, after applying the inverse transformation of phase shift $-\beta_i$, the phase shift in the i^{th} channel is $(m-1)\beta_i$.

[1] [3] incorrectly arrives at a different conclusion.

Here, we will assume that the amplitude of the nonlinear distortion is identical in all of the channels. Based on our experience, this is a good assumption for harmonics, but it is not very accurate for mixer MxN products. We are interested in how the signals in the N channels add together. Given our assumptions about the phase and amplitudes, this is equivalent to the problem of adding up N points that are uniformly distributed on a unit circle. The magnitude of the resultant vector is a random variable ranging from zero to N. This is the classic problem addressed by Rayleigh. The probability distribution function for the sum of N random points on the unit circle is given exactly by

$$p(N;r) = r\int_0^\infty x dx J_0(rx)[J_0(x)]^N \approx \frac{2r}{N}e^{-r^2/N}$$

which becomes the Rayleigh distribution [4] as shown for large N.

The noise from all channels will have a voltage gain of $\sqrt{N}$ in the beamformer, while the voltage gain on the distortion product will range from 0 to N. We now define the degradation relative to the noise power expected from N channels as $D = 20\log(\rho^2/N)$, where ρ is the voltage gain on the distortion product. The probability that the degradation will be larger than D is $P(N;\rho > r) = \int_\rho^N dr p(r;N)$ which is $e^{-\rho^2/N}$ for the Rayleigh distribution. Fig. 2 shows the degradation probability for both the Rayleigh distribution and the exact distribution for $N = 13$. The Rayleigh distribution serves as a worst-case bound, and is a good approximation away from $D|_{\rho=N}$. The allowable probability must be determined as part of a system design, but a maximum degradation not much greater than 10 dB should be safe, *independent* of the size of the array. We expect mitigation of nonlinear distortion by random phase shifts to be useful in fairly large digital arrays, i.e. when $N \geq 100$.

IV. MEASUREMENTS

We constructed a small multi-channel receive-only testbed to test our hypotheses about the effect of phase shifts on nonlinear distortion. A block diagram is shown in Fig. 3. The testbed consisted of 13 digital receivers, operating in the UHF band. Nominally, RF signals are downconverted to a common IF (10 MHz), sampled, and then digitally converted to baseband (using digital quadrature sampling).

During testing, CW tones from a frequency synthesizer are split 13 ways and sent into 13 separate LNAs that amplify the signals. The first channel serves as a reference. Two-stage analog downconverters translate the input signals to an output frequency of 10 MHz. A COTS VME board [5] performs the A/D and DDC functions.

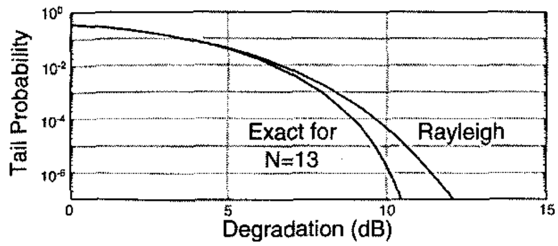

Fig. 2. Degradation relative to gain on noise.

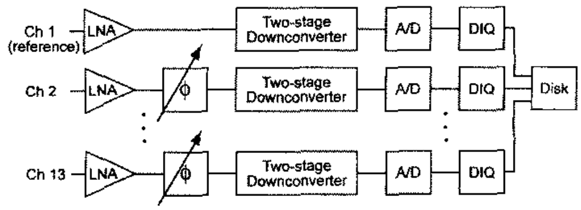

Fig. 3. Block diagram of testbed.

For both the harmonic and mixer spurious signals, we verified that the linear and nonlinear terms have a different shift relationship at the output than at the input, thus confirming that mitigation via phase shifts is possible. In our configuration, the G_i transformation is a physical transformation enacted with the phase shifters; the inverse transformation H_i is implemented in the beamformer weights in post-processing.

A. *Mitigation of –2x2 Mixer Spurs*

As discussed previously, *MxN* mixer spurs shifted at the RF input by β_i should exhibit (M-1) β_i phase shift after the G_i and H_i transformations. To test this, we considered the –2x2 spur, in which we expect phase shifts twice as large as those on the input signal.

For this test we used only two channels. Fig. 4 displays the relationship between the phase shift of the –2x2 mixer spur and the applied phase shift. As seen in the figure, the –2x2 mixer spur phase shift was 2.0138 times the applied phase shift. This closely matches the ideal phase shift of twice the input signal, confirming our assumption of the *MxN* phase relationship for the –2x2 case.

To test –2x2 mixer spur mitigation, we set the phase shifters to simulate signals coming broadside to the array and applied random phase shifts to channels two through twelve. Combining thirteen receivers produces a voltage gain of 13, so the beamformer output should be 22.26 dB larger than a single receiver's output. Therefore, 22.26 dB was added to the spectral power of the reference (Channel 1) so the beamformer output should exhibit comparable signal power. When we beamformed the channel data in the broadside tests, we found that the signal and –2x2 mixer spur both added up coherently. However, with

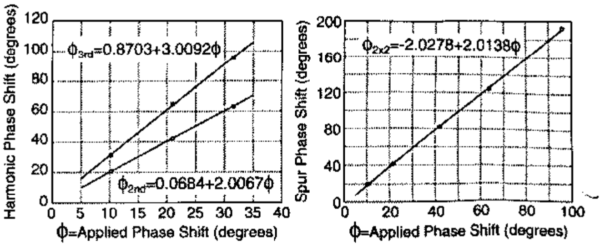

Fig. 4. The second and third harmonics exhibit the expected phase shift (left), as does the -2x2 mixer spur (right).

random phase shifts, the beamformer output at the spur decreased substantially; in most cases, the output at the spur dropped more than 11.13 dB, indicating successful phase shift mitigation of nonlinearities. Fig. 6 displays the beamformed and reference outputs for a representative test using a set of random phase shifts. The beamformed −2x2 mixer spur is 14.58 dB below the reference channel.

B. *Mitigation of Harmonic Distortion*

As discussed previously, all harmonic nonlinearities should exhibit $(M-1)\beta_i$ phase shift after the G_i and H_i transformations. We tested this for both second and third order harmonics.

Fig. 4 displays the phase relationship for both the second and third harmonic tests. The second harmonic phase shift was 2.0066 times the applied phase shift, while the third harmonic experienced a phase shift 3.0092 times the applied phase shift. This confirms our analysis of the fundamental-harmonic phase relationship.

To demonstrate the mitigating effect of phase shifts on harmonic correlation in beamforming, we beamformed the receiver data with random phase shifts at each channel input for the second harmonic test case. We consider mitigation of nonlinear distortion to be successful if the harmonic component grows more slowly than the noise. In three out of four sets of random phase shifts, the harmonic component dropped more than 11.13 dB SNR gain from beamforming. Fig. 5 displays the beamformed and reference outputs for a representative test using a set of random phase shifts. The beamformed harmonic is 14.65 dB below the reference channel.

V. CONCLUSION

We have shown that simple phase shifts can be used to mitigate the beamforming gain of certain nonlinear distortions in digital arrays. We showed this is the case for harmonic distortion and mixer *MxN* products, but not third order intermodulation distortion. We also defined the degradation relative to SNR gain in terms of the statistics of sums of random phase shifted signals. This analysis shows how mitigation via random phase shifts will perform in digital arrays.

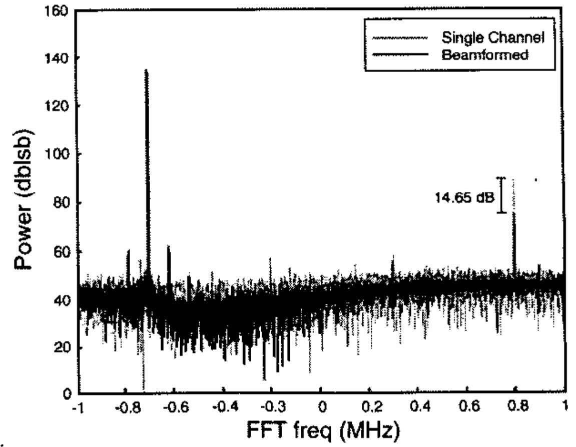

Fig. 5. Mitigation of second harmonic by random phase shifts.

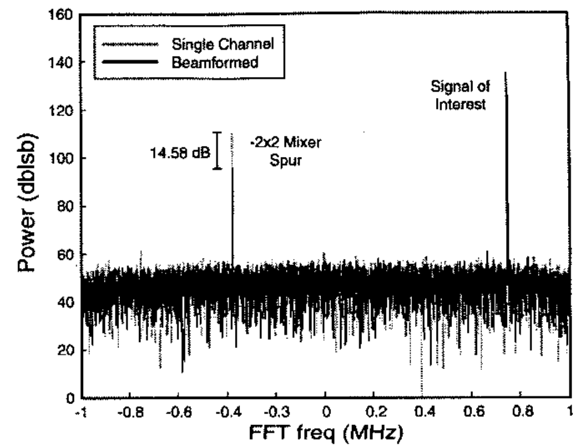

Fig. 6. Mitigation of -2x2 spur by random phase shifts.

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